

Mathematics

Advanced GCE

Unit **4733**: Probability and Statistics 2

Mark Scheme for June 2012

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations

Annotation	Meaning
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (OCR) Statistics strand

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded.
It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be an answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. These must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award the mark according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved), consult your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. It may be lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to show the intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in the question by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark will be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless a Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct answer. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where the candidate follows through the correct answer as part of a wrong argument.

- d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has done well in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, if all the steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect work. Otherwise A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) mark is given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, it is up to the examiner to decide whether to accept various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will carry through. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not in the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given. Significant figures are expected (either because of an explicit instruction or because the general context of a problem does not specify otherwise). If 3 significant figures are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a numerical value of 0.6417 is given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, an answer that is grossly over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example, where the value of a test statistic is (say) 2.12888446667 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to answers that are given as final stages of calculations; intermediate working should usually be carried to a greater degree of accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the scheme rationale. If in doubt, contact your Team Leader.

- ## g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examine the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

- h Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object being considerably changed, it is likely that all the marks for that question, or section of the question, will be often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but all marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even when difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps as a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so slight an effect as to be the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be more than 1 mark. In sections of questions where there is a numerical answer either at the end of the section or to be obtained and communicated (e.g. the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark must be withheld even if all marks have been earned. It will also often be the case that such a mark is implicitly "cao" even if not explicitly designated as such.

On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-through for a misread of the value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the candidate retains knowledge of how to compare it with a critical value and draw conclusions. Such "fresh starts" are not affected by any errors made earlier.

A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. These are more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they do not, they should be marked as possible in the same way as numerical misreads, *mutatis mutandis*. This also applies to misreads of text, which are likely to cause major problems in fair marking.

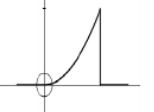
The situation regarding any particular cases that arise while you are marking for which you feel you need detailed guidance should be discussed with your Team Leader.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer	Marks	Guidance
1		Number CDs (sequentially) Select using random numbers	B1 B1 [2]	List needn't be stated, but must mention CDs. <i>Not</i> "select numbers randomly". Hat, etc: B1B0 Systematic: 66 or 67 B1, random start B1 Assume s. un... If "number C" "sort by number"
2	(i)	$\left(\frac{71.2 - 72.0}{\sigma / \sqrt{40}} \right) = -0.3853$ $[\sigma = 13.13,] \text{Var}(V) = 172.4$	M1 A1 B1 A1 [4]	Standardise with Φ^{-1} & $\sqrt{40}$, allow cc, \checkmark errors eg σ^2 Square roots and sign correct, no cc, no "1 -" error z in range (\pm) [0.385, 0.386] seen <i>Final</i> answer in range [172, 173], or 13.1^2 cwo RHS must be Φ^{-1} , i.e. 0.6368 or 0.35. "1 -" [0.674 may be from " Needs variance, not S. NB: Look out for -13.
2	(ii)	Parent distribution not known n is large	B1 B1 [2]	Or clear equivalent. Not "sample not normal" Or clear equiv, e.g. sample size > 30 . Extras: max 1 "n large, $n > n_0$ ": B1 if $n_0 \geq 30$. Don't bother about or... If numerical must be 3
3		$\alpha:$ $H_0: p = \frac{1}{3}$ [or 0.33 or better] $H_1: p \neq \frac{1}{3}$ [or 0.33 or better] $B(12, \frac{1}{3})$ stated or implied $P(\geq 7) = 1 - 0.9336 = 0.0664 > 0.025$	B2 M1 A1 A1 [7]	Allow π , but $\mu = \frac{1}{3}$ etc B1. Any other letter, B0 One-tailed, or no symbol, B1 max $B(12, \frac{1}{3})$ stated or implied, allow for $N(4, 8/3)$, Po(4) Probability in range [0.066, 0.067] Explicit comparison with 0.025, or $2p$ with 0.05 Not $\mu = 4$ <i>(if in doubt, consult)</i> <i>If N used, or P(≤ 7) or</i> 1-tailed: A0 here regard
		$\beta:$ CR is ≥ 8 , 7 not in CR Probability is 0.0188	A1 A1 [7]	Needs explicit comparison of 7 with CV Must be ≥ 7 , 0.019 or 0.0188 or better, allow 0.9812 Need to be clear that C for comparison with 7
		Do not reject H_0 . Insufficient evidence that statement is false.	M1 A1 \checkmark [7]	Needs correct method, including like-with-like, correct tail, ≥ 7 (or ≤ 6). If CV, needs right tail A1 needs "evidence" or equivalent. "Statement" is enough context here Allow from 1-tail. 0.9 M0 unless " ≥ 7 " stated \checkmark on their p/CR. With only to p .

Question		Answer	Marks	Guidance
4	(i)	Crystals must occur independently of one another	B1 [1]	Allow interpreted, or “randomly” but nothing else. Must be contextualised; no other answers included.
4	(ii)	$e^{-3.2} \frac{3.2^5}{5!} = 0.114(0)$	M1 A1 [2]	Formula, or .0608 or .1781 or .1075 or .1203 (tables) Answer a.r.t. 0.114, implies both marks
4	(iii)	$\text{Po}(2.368)$ $1 - e^{-2.368} (1 + 2.368 + \frac{2.368^2}{2})$ $= 0.4219$	M1 M1 A1 [3]	Po(0.74 × 3.2) stated or implied 1 – correct Poisson terms, their λ , allow ± 1 term Answer, a.r.t. 0.422, implies all 3 marks
4	(iv)	$\text{Po}(32) \approx N(32, 32)$ $1 - \Phi\left(\frac{35.5 - 32}{\sqrt{32}}\right)$ $= 1 - \Phi(.619) = 0.2681$	M1 A1 M1 A1 A1 [5]	$N(\lambda, \lambda)$ stated or implied, allow $\sqrt{\lambda}$ or λ^2 for var $N(32, 32)$, allow $\sqrt{32}$ or 32^2 for var Standardise with λ and $\sqrt{\lambda}$ or λ , allow cc errors but not \sqrt{n} ; both cc and $\sqrt{ }$ correct Final answer, a.r.t. 0.268

Question		Answer	Marks	Guidance
5		$H_0: \mu = 6.1$ $H_1: \mu \neq 6.1$ $\hat{\mu} = \bar{x} = 6.2$ $\hat{\sigma}^2 = \frac{80}{79} \left(\frac{3126}{80} - 6.2^2 \right) = 0.643$ $z = \frac{6.2 - 6.1}{\sqrt{0.643/80}} = 1.115$ $[1 - \Phi(1.115) = 0.1325 > 0.05]$ $1.115 < 1.645$	B2 B1 M1 M1 A1 M1 A1 A1	Both: B2. One error, B1, but \bar{x} , x , r etc: 0. 6.2: B0 6.2 [31/5] seen somewhere (other than hypotheses) Correct formula for biased estimate [0.635 or 127/200] Divide by 79 somewhere Variance estimate, a.r.t. 0.643, can be implied Standardise their 6.2 with reasonable variance attempt, needs 80, allow cc $z \in [1.11, 1.12]$ (not $-$) or $p \in [0.1323, 0.1333]$ Compare z with 1.645 (allow -1.645 if $z < 0$) or $p (< 0.5)$ with 0.05
		$\beta:$ $CV \ 6.1 + 1.645 \times \sqrt{\frac{0.643}{80}}$ $= 6.247 \text{ and } 6.2 < 6.247$	M1 A1 A1 \checkmark	$6.1 + z\sqrt{(\sigma^2/80)}$, allow \pm , \checkmark errors CV, a.r.t. 6.25, needs $z = 1.645$, allow biased $\hat{\sigma}^2$ Compare 6.2 with CV from + sign, \checkmark on z (but not σ)
		Do not reject H_0 . Insufficient evidence that pH value is not 6.1	M1 A1 \checkmark [11]	Needs essentially correct method and comparison, needs 80 but no need for correct variance Needs context and “evidence” or equivalent, ft on their $z/p/CV$
Notes:		Biased estimate used : typically gets B2B1 M1M0A0 M1A0A1 M1A1 [total 8]		\bar{x} and μ interchanged: allow final M1A1 if anywhere right, but if always wrt M0A0. This would typically get B0B0B1 M1M1A1 M1A0A0 M0A0 [total 5]

Question		Answer	Marks	Guidance	
6	(i)	B(32, 0.4) ≈ N(12.8, 7.68) Valid as 12.8 and 19.2 > 5 $1 - \Phi\left(\frac{17.5 - 12.8}{\sqrt{7.68}}\right)$ [= 1 - Φ(1.696)] = 0.0449	B1 M1A1 B1 M1 A1 A1 [7]	B(32, 0.4) stated or implied, e.g. by Po(12.8) N(their attempt at np , npq); N(12.8, 7.68) Or “ n large and p close to 0.5”. Not npq or 7.68 > 5. Standardise, their np , npq , allow wrong/no cc or no ✓ 17.5 and \sqrt{npq} correct Answer, a.r.t. 0.045	Poisson [1] or SC: B(12.8, 7.68) Allow np and n , ÷32: M0
6	(ii)	B(90, 0.01) ≈ Po(0.9) $e^{-0.9} \frac{0.9^3}{3!} = \mathbf{0.0494}$	B1 M1 M1 A1 [4]	B(90, 0.01) stated or implied. Po(their attempt at np) Correct formula or use of tables, e.g. 0.1646 or 0.0112 Final answer in range [0.049, 0.05) [i.e., not 0.05]	Exact [0.049003]: B1 Don't treat $p = 0.1$ as 1 No working, wrong answer ⇒ M1A1 SC: B(90, 0.1), N(9, 8) two B2
7	(i)		M1 A1 B1 [3]	Positive parabola (only), through 0, nothing below x -axis Clear truncation at ends Withhold if concept misunderstood. Need to have probability of <i>values</i> (not of <i>occurring</i>); not just shape. Allow for U-shape but nothing else	$k < 0$: M0 even if $k > 0$ Don't need any scales. Can be vertical at A , not O . E.g.: “More likely to a ” B0. Ignore extra comments
7	(ii)	(a) $\int_0^a kx^2 dx = 1 \Rightarrow k = \frac{3}{a^3}$ $\int_0^a \frac{3}{a^3} x^3 dx = \frac{9}{2} \Rightarrow a = 6$	M1 A1 M1 A1 A1 A1 [6]	Attempt to integrate kx^2 , ignore limits Correct limits and equate to 1 Attempt to integrate kx^3 , ignore limits Correct limits and equate to 4.5 One correct equation connecting k and a , can be implied Correctly obtain $a = 6$ only	Must attempt integration Must attempt integration Don't need k in terms of a . $ka^3 = 3$ or $ka^4 = 18$, a.c. No marks explicitly for $a = 6$ [0.01388...]

Question			Answer	Marks	Guidance	
7	(ii)	(b)	$\int_0^6 \frac{1}{72} x^4 dx = \frac{108}{5}$ $21.6 - 4.5^2 = 1.35$	M1 A1 A1 [3]	Attempt to integrate kx^4 , their a, k , can be algebraic Subtract 4.5^2 (given in question) 1.35 or exact equivalent only	Must attempt Somewhere [=27/20]
8	(i)		$30 + 1.645 \times \frac{8}{\sqrt{18}} = 33.102$ so CR is $\bar{X} > 33.1$	M1 A1 A1 A1/ [4]	$30 + z \times 8/\sqrt{18}$, allow \sqrt errors, cc 1.645, requires + only 33.1 a.r.t. 33.10 \geq their RH CV \sqrt , allow \leq their LH CV as well, allow $>$, allow no letter or X but no other letter	Allow \pm but not – only Don't allow "accept i" Inequality required in
	(ii)		Type I [error]	B1 [1]	Nothing else unless it's just an amplification. Allow "Type 1"	
	(iii)		B(20, 0.05): $P(\geq 4) = 0.0159$ so unlikely that $\mu = 30$	M1 A1 A1/ [3]	B(20, 0.05) stated or implied. Not B(20, 1/5) Probability, a.r.t. 0.016 Justified conclusion, e.g. "I think $\mu = 30$ as not less than 0.01". FT on their p .	No reason: A0. Not or think $\mu = 30$ as probab
	(iv)		$\frac{33.1 - \mu}{8/\sqrt{18}} = -0.253$ $\mu = 33.58$	M1 A1 A1 [4]	Needs Φ^{-1} , their CV, SD right or same as in (i), allow cc Signs correct, can be implied by answer $>$ their CV z in range (\pm)[0.25, 0.26] Final answer $33.55 \leq \mu \leq 33.60$, 4 SF needed.	Not 30. Allow omission in (i). "1 –" errors: can Typically 32.62 probab

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